

# TEMPERATURE DEPENDENCE OF GRAPHENE TRANSPORT COEFFICIENTS

S. K. Jaćimovski, D. I. Raković

Belgrade, Serbia

## INTRODUCTION

General expressions for energy dependencies of transport coefficients are obtained by solving the Boltzmann transport equation. The paper presents their temperature dependencies in graphene. Coefficient of electrical conductivity, coefficient of electronic thermal conductivity, Seebeck coefficient and Lorentz function in graphene were analyzed. The expressions for all transport coefficients were found analytically in the low temperature range  $\sim 50$ -100 K, and then presented graphically as well.

## TRANSPORT COEFFICIENTS OF GRAPHENE

By solving Boltzmann transport equation in the approximation of relaxation time, general form for transport coefficients are:

$$L_{ij}(\varepsilon) = (-1)^{i+j} \int_0^{\infty} D(\varepsilon) \tau(\varepsilon) (\varepsilon - \mu)^{i+j-2} \left( -\frac{\partial f}{\partial \varepsilon} \right) d\varepsilon; \quad i, j \in (1, 2)$$

where  $L_{ij}(\varepsilon)$  are transport coefficients,  $D(\varepsilon)$  is density of states of charge carriers,  $\tau(\varepsilon)$  is relaxation time of the system,  $\mu$  is chemical potential of charge carriers,  $f(\varepsilon)$  is Fermi-Dirac distribution of charge carriers, and  $\varepsilon$  is energy of charge carriers.

$L_{11} = \sigma$  is coefficient of electrical conductivity,

$S = \frac{1}{|e|T} \frac{L_{12}}{L_{11}}$  is coefficient of thermo-electromotor force,

$\kappa = \frac{1}{|e|T} \frac{L_{22}}{L_{11}} - \frac{L_{12}^2}{L_{11}^2}$  is coefficient of electronic thermal conductivity,

$L = \frac{\kappa}{T\sigma}$  is Lorentz coefficient.

## ANALYTICAL EXPRESSIONS FOR TRANSPORT COEFFICIENTS IN CONSTANT ELECTRIC FIELD

It is shown in literature that for most experimentally measured temperature dependencies of transport coefficients, there should be adopted relaxation time for interactions of charge carriers on long-range Coulomb potential. Hereby we consider graphene monolayer on  $\text{SiO}_2$  substrate. For this case relaxation time for interactions on long-range Coulomb potential is:

$$\tau = \Lambda \cdot \varepsilon; \quad \Lambda = 7.1 \cdot 10^5 \text{ s/J}$$

Coefficient of electrical conductivity is given by

$$\sigma = \sigma_{\min} \frac{\Lambda}{\hbar} \int_0^{\infty} \varepsilon^2 d\varepsilon \cdot \frac{e^{-\frac{\varepsilon-\mu}{kT}}}{\left( e^{-\frac{\varepsilon-\mu}{kT}} + 1 \right)^2}; \quad \sigma_{\min} = 2e^2/h$$

where  $e$  is elementary electrical charge,  $h$  is Planck constant, and  $\mu$  is chemical potential. By substitution  $x = \frac{\varepsilon-\mu}{kT}$ , limits of integration are  $x_d = -\frac{\mu}{kT}$ ;  $x_g = \infty$ . As  $\frac{\mu}{kT} \gg 1$  (low-temperature development) lower limit can be replaced by  $x_d = -\infty$ .

As a result, following expression for scaling coefficient of electrical conductivity in constant electric field is obtained:

$$\frac{\sigma}{\sigma_{\min}} = \frac{\Lambda}{\hbar} \left( \mu^2 + \frac{\pi^2}{3} (kT)^2 \right)$$

By similar procedure we find expressions for other kinetic coefficients:

$$S = \frac{1}{|e|T} \frac{\frac{2}{3} \pi^2 \mu (kT)^2}{\mu^2 + \frac{\pi^2}{3} (kT)^2}$$

$$\kappa = \frac{1}{e^2 T} \left\{ \frac{\Lambda}{\hbar} \left( \frac{\pi^2 \mu^2 k^2 T^2}{3} \right) + \frac{7\pi^4}{15} k^4 T^4 - \frac{4\pi^4}{9} \frac{k^4 T^4}{\mu^2 + \frac{\pi^2}{3} k^2 T^2} \right\}$$

$$L = \frac{1}{e^2 T} \left\{ \frac{\Lambda}{\hbar} \left( \frac{\pi^2 \mu^2 k^2 T^2}{3} \right) + \frac{7\pi^4}{15} k^4 T^4 - \frac{4\pi^4}{9} \frac{k^4 T^4}{\mu^2 + \frac{\pi^2}{3} k^2 T^2} \right\} \frac{1}{T \left( \sigma_{\min} \frac{\Lambda}{\hbar} \left( \mu^2 + \frac{\pi^2}{3} k^2 T^2 \right) \right)}$$

## ANALYTICAL EXPRESSIONS FOR TRANSPORT COEFFICIENTS IN ALTERNATE ELECTRIC FIELD

If alternate electric field  $E = E_0 \cos \Omega t$  is applied to graphene sample, expressions for transport coefficients have the following forms:

$$\frac{\sigma_{\Omega}}{\sigma_{\min}} = \frac{\frac{\Lambda}{\hbar} \left( \mu^2 + \frac{\pi^2}{3} (kT)^2 \right)}{1 + (\Omega \Lambda \mu)^2}$$

$$S_{\Omega} = \frac{2}{3} \frac{\pi^2}{eT} \frac{(kT)^2}{\mu} \frac{1 + (\Omega \Lambda \mu)^2}{\mu^2 + \frac{\pi^2}{3} (kT)^2}$$

$$\kappa_{\Omega} = \frac{2}{3} \frac{\pi^2}{hT} (kT)^2 \mu^2 - \frac{8}{9} \frac{\pi^4}{hT} \frac{(kT)^4}{\mu} \frac{1 + (\Omega \Lambda \mu)^2}{1 + \frac{\pi^2}{3} \frac{(kT)^2}{\mu^2}}$$

$$L_{\Omega} = \left\{ \frac{2}{3} \frac{\pi^2}{hT} (kT)^2 \mu^2 - \frac{8}{9} \frac{\pi^4}{hT} \frac{(kT)^4}{\mu} \frac{1 + (\Omega \Lambda \mu)^2}{1 + \frac{\pi^2}{3} \frac{(kT)^2}{\mu^2}} \right\} \frac{1 + (\Omega \Lambda \mu)^2}{\sigma_{\min} \frac{\Lambda}{\hbar} \left( \mu^2 + \frac{\pi^2}{3} (kT)^2 \right)}$$

For numerical calculations, we adopt the following values of parameters:

$$\mu = 0.8 \cdot 10^{-22} \text{ J}; \quad \varepsilon_r = 2.4; \quad \Omega = 10^{12} \text{ Hz}; \quad \Lambda = 7.1 \cdot 10^5 \text{ s/J}$$

## TEMPERATURE DEPENDENCIES OF TRANSPORT COEFFICIENTS

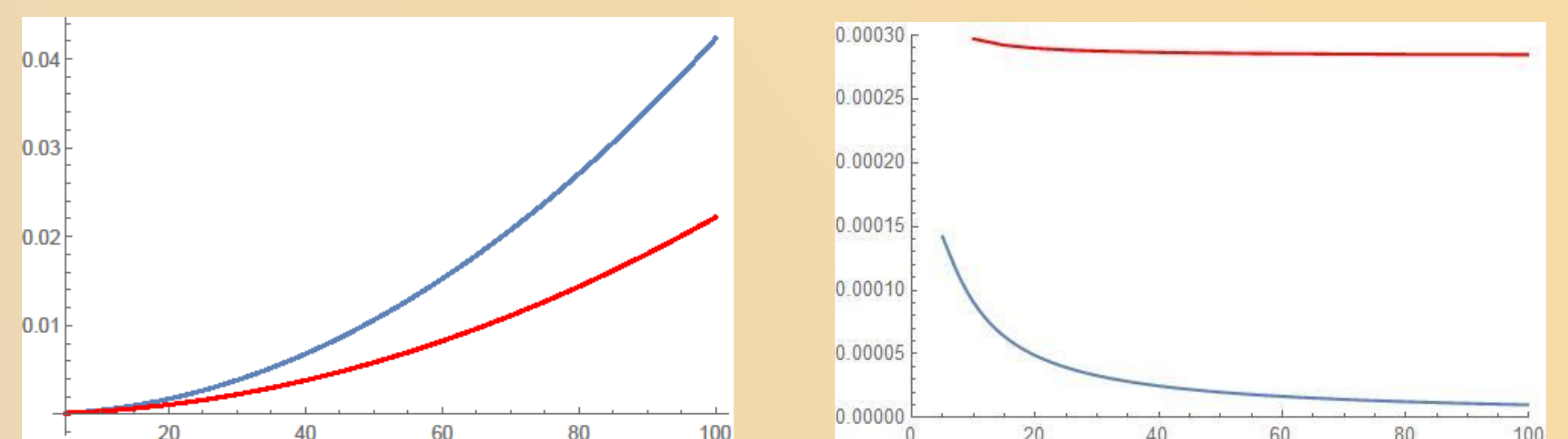


Figure 1. (left) Temperature dependencies of normalized electrical conductivity; (right) Temperature dependencies of Seebeck coefficient. In both cases, blue line represents analytical dependencies & red line numerically obtained dependencies.

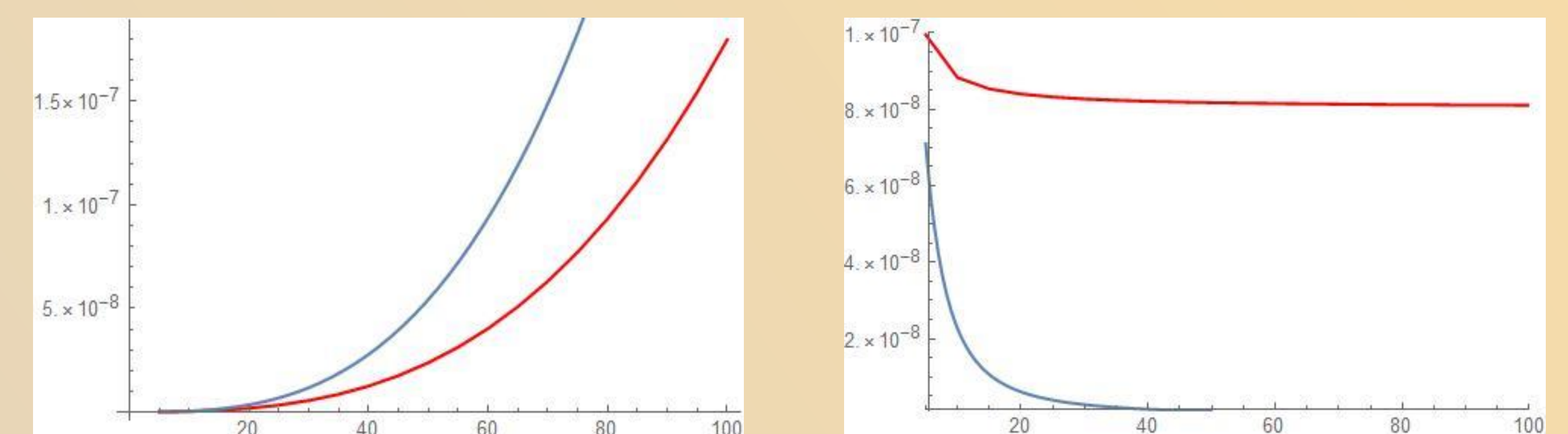


Figure 2. (left) Temperature dependencies of coefficient of electronic thermal conductivity; (right) Temperature dependencies of Lorentz coefficient. In both cases, blue line represents analytical dependencies & red line numerically obtained dependencies.

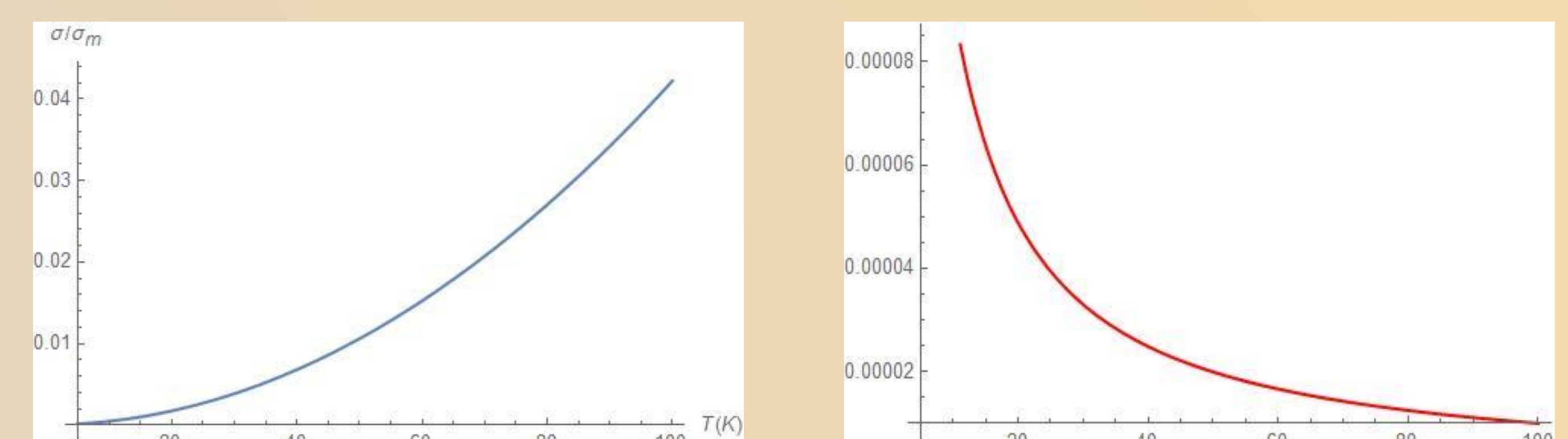


Figure 3. Temperature dependencies of transport coefficients for case of alternate electric field applied.

## CONCLUSION

The paper presents energy & temperature dependencies of transport coefficients (electrical conductivity, electronic thermal conductivity, Seebeck coefficient, Lorentz function) in constant & alternate electric fields in graphene. The expressions for them were found analytically in approximation of relaxation time in low temperature range  $\sim 50$ -100 K, and presented graphically as well.