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Face Recognition by Using Unitary Vector Spaces

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***Abstract*—Dynamic developments of science and technology have demanded necessity of interdisciplinary approach and appearence of novel scientific disciplines. In this respect, face recognition using quantum mechanical methods of unitary vector spaces, represents very interesting field due to possible applications in the field of quantum informatics. Thus traditional quantum mechanical methods widely applied to microsystems during the past century, are now succesfully extrapolated in macroscopic information framework as well.**

***Keywords*—Quantum Associative Neural Networks; Face Recognition.**

I. Introduction

THE subject of this paper is possibility of face recognition out of existing set of pictures by computer simulations in quantum-mechanical framework, through realized analogy between unitary vector space and set of pictures. This work was inspired by basic investigations in this field [1-7], and possible applications as well. The main idea of these previous papers is possibility of object's pattern reconstruction on the basis of set of templates, quantum interference and associative quantum neural networks. The term "neural" is not quite adequate, but in formal sense it can be accepted as applied quantum-holographic procedure resembles on classical Hopfield neural network [8-10].

The potential of quantum neural networks in the field of pattern recognition is great, because of very fast parallel process expressed through fundamental phenomenon of *wave function collapse* on the very one eigenfunction out of basis set of eigenfuntions. The only limitation of these networks is based on the computer hardware.

In this paper the action of *projector in the basis of unitary vector space* reconstructs input photo, by using associative effects of quantum-mechanical interference and developing the representing wave function over corresponding basis set of eigenfuntions.

II. Mathematical Model

The input vectors, whose dimensionality is defined by number of pixels $\nu_j^{\,k}(j=1,2,...,N)$, written in Dirac notation have the following form:

$$\left|k\right\rangle = (\nu_1^{\,k}, \nu_2^{\,k}, ..., \nu_N^{\,k}) . \qquad (1)$$

In order to define complete vector space it is necessary to define the scalar product, and the adopted scalar product of the form

$$\left\langle k \,\middle|\, q \right\rangle = \sum_{ij}^{N} \nu_i^{\,k^*} \nu_j^{\,q} \qquad (2)$$

satisfies all the axioms of unitary vector space, because it has very wide application in quantum mechanics. The correspondence between the set of photos and the unitary vector space cannot be completely established before defining the orthogonality of vectors i.e. pictures. From the very proposed problem, it is clear at first glance that for standard values of the pixels $\nu^k \in [0,255]$ orthogonality needs not to be fulfilled, and hence the intensity of the input vectors are "mapped" within interval $\nu^k \in [0,1]$. In mathematical procedure of photos' preprocessing, applied by Peruš *et al.* [3], the value of every pixel is calculated as follows:

$$\bar{\nu}_i^{\,k} = \nu_i^{\,k} - \frac{1}{N}\sum_{i=1}^{N} \nu_i^{\,k} \qquad (3)$$

for all values of indices *i,k*, and afterwards the obtained vectors are normalized to unity,

$$\sum_{j=1}^{N} (\bar{\nu}_j^{\,k})^2 = 1 , \qquad (4)$$

and finally "centered" by applying relation:

$$\sum_{j=1}^{N} \bar{\nu}_j^{\,k} = 0. \qquad (5)$$

In the case of high-resolution photos, the condition of orthogonality is fulfilled, and further procedures of orthonormalization are not even necessary. In the vector space with scalar product (2), the *projector* has the following form:

$$P = \sum_k \left|k\right\rangle\left\langle k\right| = \sum_{k=1}^{p}\left(\sum_{ij}^{N} \nu_i^{\,k^*} \nu_j^{\,k}\right). \qquad (6)$$

The action of projector on a vector from orthonormalized basis produces this same vector, while an arbitrary vector from the unitary space is "projected" on the basis according to:

$$P\left|q\right\rangle = \sum_k \left\langle k \,\middle|\, q\right\rangle \left|k\right\rangle = \sum_k c_{k,q}\left|k\right\rangle . \qquad (7)$$

In matrix representation, the eq. (6) can be rewritten as:

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$$P = \sum_{ij}^{N} \left(\sum_{k=1}^{p} \nu_i^{\,k} \otimes \nu_j^{\,k} \right) = \sum_{ij}^{N} G_{ij} \; ; \qquad (8)$$

where $G_{ij} = \sum_{k=1}^{p} \nu_i^{\,k} \otimes \nu_j^{\,k^*}$ is the sum of autocorrelation functions of the input object. This relation is equivalent to Hebbian matrix element in classical Hopfield neural nets.

III. Experimental Data

The investigation of applicability and validity of eqs. (1-8) was realized by numerical simulations in the MATLAB environment, on the set of photos presented in figs. 1-6:

fig.1

fig.4

fig.2

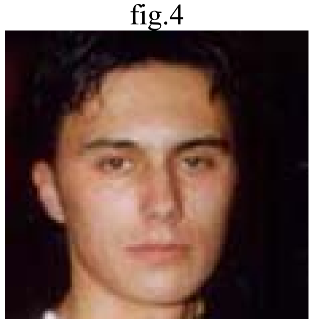

fig.5

fig.3

fig.6

On this basis, it is possible to find probability of "recognition" of the photos (of the same person or different ones) according to relation: $C_{k,q} = \langle k | q \rangle$. In this case, the vectors corresponding to the photos in figs. 1, 4, 5, and 6 represent orthonormalized basis of the created (4D) unitary vector space. The calculated coefficients $C_{k,q}$ of mutual "recognition" of the photos in figs. 1-6. are listed in table 1.

By analysis of the results from the table, it can be concluded that photos of the same person with different facial expressions can be mutually related with probability of over 77% (these results being bolded out in the table), which can be satisfactory for eventual practical applications. However, different positions of the person's enface in the photos (in a sense of rotations and translations) can produce inadequate results, so that geometrical transformations must be applied to pixel coordinates of photos from the given set. This will be the subject of the further investigation and the test of practical applicability of the applied method as well.

Table 1. The coefficients $C_{k,q}$ of mutual "recognition" of the photos in figs. 1-6.

$C_{k,q}$	1	2	3	4	5	6
1	**1.00**	**0.83**	**0.77**	0	0	0
2	**0.83**	**1.00**	**0.88**	0.1	0.11	0.04
3	**0.77**	**0.88**	**1.00**	0.06	0.1	0.06
4	0	0.1	0.06	1.00	0	0
5	0	0.11	0.1	0	1.00	0
6	0	0.04	0.06	0	0	1.00

IV. Conclusions

In this paper the possibility of face recognition of some person from the chosen set of photos is analyzed by using quantum mechanical methods of unitary vector spaces. The obtained results imply that the proposed procedure functions in the manner of neural networks, which means that input vector slightly differs from the values obtained by neural network training (i.e. the basis vectors). Regardless on this limitation, this method can be rather efficient having in mind great speed of parallel processing.

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